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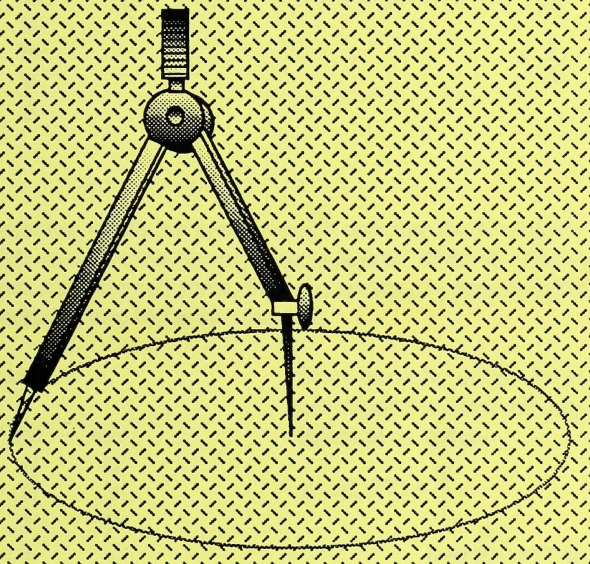
MATHEMATICS 23

LEARNING FACILITATOR'S MANUAL

UNIT 8



Distance Learning

Alberta EDUCATION

Note

This Mathematics Learning Facilitator's Manual contains answers to teacher-assessed assignments and the final test; therefore, it should be kept secure by the teacher. Students should not have access to these assignments or the final tests until they are assigned in a supervised situation. The answers should be stored securely by the teacher at all times.

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Topic 1: Probability

(5) 1. Conduct an experiment which involves choosing at random 3 balls from a box, making sure to record the color and the order in which they were chosen. How many possible outcomes are there if the box contains 2 red, 1 green, and 2 blue balls? That is, indicate how many times you feel you will need to repeat the experiment to ensure that every combination is likely to be selected. From the recorded results of your experiment, determine the statistical probability of choosing the ordered combination of balls blue-green-blue.

Solutions will vary.

1st	2nd	3rd
R	R	G
R	R	B
R	G	B
R	B	B
G	B	B
G	R	B
. . . .		

Since the order of the balls is important, at least 60 draws will be required.

$$P(\text{blue, green, blue}) \doteq \frac{2}{60}$$
$$\doteq \frac{1}{30}$$

(5) 2. Conduct an experiment which involves tossing a coin into the air 4 times and recording, in order, how it lands. You should repeat the experiment at least 20 times. Determine how many different outcomes there are and show, from the results of your experiment, the statistical probability of obtaining two heads and two tails.

Solutions will vary.

1st	2nd	3rd	4th
H	H	H	H
H	H	H	T
H	H	T	T
H	T	T	T
T	T	T	T
H	T	H	T
. . . .			

Since the order of the tosses are not important, there are only 5 different outcomes.

$$P(2H, 2T) \doteq \frac{3}{8}$$

(5)

3. Conduct an experiment which involves using only the kings and the queens from a deck of playing cards. From these 8 cards, deal 2 cards to one player and 3 cards to another. Repeat the experiment at least 25 times. Determine how many different outcomes there are and show, from the results of your experiment, what the statistical probability is of dealing the king of hearts and a black queen to the player with 2 cards.

Solutions will vary.

1st	2nd
K♠K♣	K♦K♥Q♠
K♠K♣	K♦K♥Q♣
K♠K♣	K♦K♥Q♦
K♠K♣	K♦K♥Q♥
K♠K♣	K♦Q♠Q♣
K♠K♣	K♦Q♠Q♦
. . . .	

There are 560 different outcomes.

$$P(\text{king hearts, queen black}) \doteq \frac{40}{560}$$

$$\doteq \frac{1}{14}$$

(5) 4.

Number	Frequency Tally
2	II
3	IIII
4	IIIIIII
5	IIIIII
6	IIIIIIII
7	IIIIIIIIIIIII
8	IIIIIIIII
9	IIIIII
10	IIIIII
11	III
12	I

Two dice are rolled as part of a probability experiment and the results are recorded above. From the frequency table, determine the following. Express these as reduced fractions.

a. the probability of rolling a number greater than 4

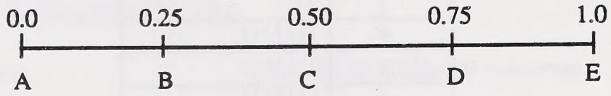
Probability of rolling a number > 4 = $\frac{54}{67}$.

b. the probability of rolling a number less than or equal to 10

Probability of rolling a number ≤ 10 = $\frac{63}{67}$.

(5) 5. Circle the correct response.

PROBABILITY LINE

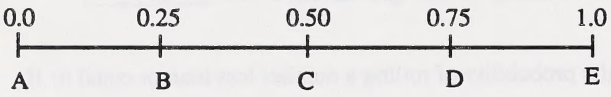


If an event is **certain** to occur, which probability represents this situation?

a. A
b. B
c. C
d. D
e. E

(5) 6. Circle the correct response.

PROBABILITY LINE



If the probability that an event will occur is equal to the probability that it will not occur, which position on the probability line represents this situation?

a. A
b. B
c. C
d. D
e. E

(5) 7. Suppose you are tossing a coin into the air and recording whether it lands heads (H) or tails (T). If you toss the coin thrice, what is the probability of recording HTT? Express your answer as a ratio in reduced fractional form.

There is only 1 way to get HTT.
There are 8 different outcomes.
(The order of tosses are important.)

P(HTT) = $\frac{1}{8}$

(5) 8. Two dice are tossed. What is the probability of tossing a number less than or equal to 10? Express your answer as a ratio in reduced fractional form.

There are 33 ways to get 10 or less.
There are 36 possible results on the dice.

P(number ≤ 10) = $\frac{33}{36}$

P(number ≤ 10) = $\frac{11}{12}$

(5) 9. A basket at the fruit stand contains 21 apples, 19 oranges, and 8 plums. If you were to close your eyes and reach into the basket to select one piece of fruit, what would be the probability of not purchasing a plum? Express your answers as a ratio reduced to lowest terms.

There are 21 + 19 or 40 items that are not plums.
There are 21 + 19 + 8 or 48 items in the basket.

P(not plum) = $\frac{40}{48}$

P(not plum) = $\frac{5}{6}$

There is a probability of $\frac{5}{6}$ of not selecting a plum.

(5) 10. Demographic Report for Huntsville

Age Class (a)	Number of People
0 - 9	156
10 - 19	194
20 - 29	152
30 - 39	155
40 - 49	124
50 - 59	107
60 - 69	74
70 +	38

While on a tour of Huntsville, suppose you said good-bye to the last person you saw. Expressing your answer in decimal form, determine the probability that the person is in one of the following categories.

a. older than 39 years of age

$$P(>39) = \frac{124+107+74+38}{1000}$$

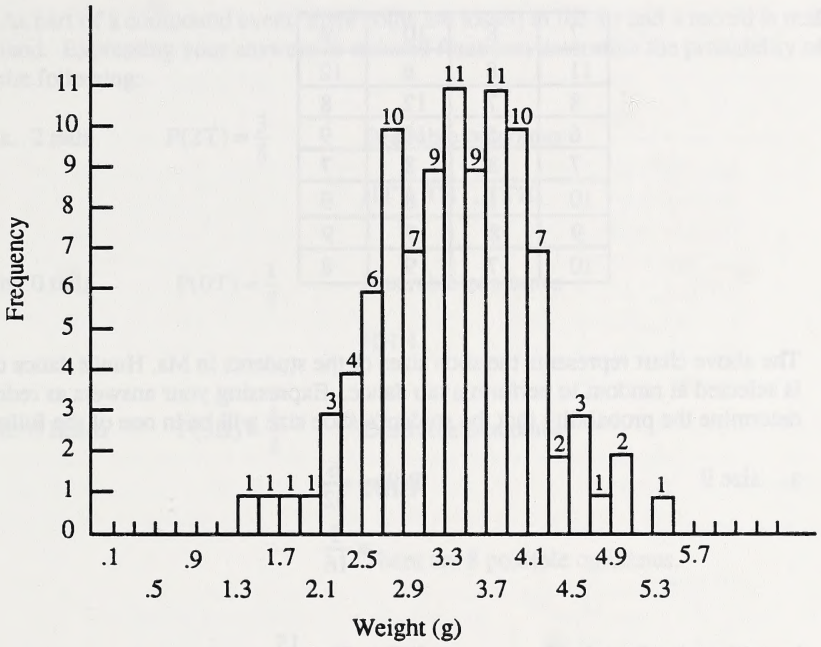
$$= \frac{343}{1000} \text{ or } 0.343$$

b. less than 50 years of age

$$P(<50) = \frac{124+155+152+194+156}{1000}$$

$$= \frac{781}{1000} \text{ or } 0.781$$

⑤ 11.



The horizontal axis of the above histogram represents the weights, in grams, of an assortment of chocolates for sale at The Olde Sweets Shoppe. Upon entering the store, you are unsure of which chocolate to purchase. You decided to blindly point in the direction of the display case to make your selection. Expressing your answers as reduced fractions or as decimals, determine the probability of choosing a chocolate weighing between the following weights.

a. 1.7 g and 2.5 g

$$P(1.7 < m < 2.5) = \frac{1+1+3+4}{100}$$

$$= \frac{9}{100} \text{ or } 0.09$$

b. 2.9 g and 3.7 g

$$P(2.9 < m < 3.7) = \frac{7+9+11+9}{100}$$

$$= \frac{9}{25} \text{ or } 0.36$$

c. 4.1 g and 5.7 g

$$P(4.1 < m < 5.7) = \frac{7+2+3+1+2+1}{100}$$

$$= \frac{4}{25} \text{ or } 0.16$$

(5) 12.

7	8	10	9
11	8	6	12
8	7	12	8
6	5	10	9
7	8	8	7
10	11	8	9
9	8	7	9
10	7	9	8

The above chart represents the shoe sizes of the students in Ms. Hunt's dance class. One student is selected at random to perform a tap dance. Expressing your answers as reduced fractions, determine the probability that the student's shoe size will be in one of the following categories.

a. size 9 $P(9) = \frac{6}{32}$

$= \frac{3}{16}$

b. either size 8 or size 9 $P(8 \text{ or } 9) = \frac{15}{32}$

c. not less than size 8 $P(\geq 8) = \frac{23}{32}$

(5) 13. As part of a compound event, three coins are tossed in the air and a record is made of how they land. Expressing your answers as reduced fractions, determine the probability of getting one of the following:

a. 2 tails $P(2T) = \frac{3}{8}$ Desirable outcomes

HTT, THT, TTH

b. 0 tails $P(0T) = \frac{1}{8}$ Desirable outcomes

HHH

c. 3 heads $P(3H) = \frac{1}{8}$ Desirable outcomes

HHH

There are 8 possible outcomes.

(5) 14. As part of a compound event, three cards are drawn from a deck of 52 playing cards. Expressing your answers as reduced fractions, determine the probability of drawing one of the following combinations.

a. the ace ♣, king ♣, and queen ♣

$$P(A\clubsuit, K\clubsuit, Q\clubsuit) = \frac{3}{52} \times \frac{2}{51} \times \frac{1}{50}$$
$$= \frac{6}{132\,600}$$
$$= \frac{1}{22\,100}$$

b. the 7 ♠, 5 ♣, and 6 ♠

$$P(7\spadesuit, 5\clubsuit, 6\spadesuit) = \frac{3}{52} \times \frac{2}{51} \times \frac{1}{50}$$
$$= \frac{6}{132\,600}$$
$$= \frac{1}{22\,100}$$

(5) 15. Two items are selected at random, first from Table 1 and then from Table 2. Determine the following probabilities of the compound event. Report your answers as reduced fractions.

Table 1

4	2	4	1
1	3	1	2
3	2	1	2
2	3	4	1
1	2	4	4
1	3	4	1
1	2	4	2

Table 2

★	o	∞	Δ
∞	o	∞	★
★	Δ	★	Δ
★	o	★	o
★	∞	∞	★
∞	o	Δ	Δ
Δ	o	∞	∞

a. Probability (2 and o)

$$P(2 \text{ and } o) = \frac{8}{28} \times \frac{6}{28}$$
$$= \frac{48}{784}$$
$$= \frac{3}{49}$$

b. Probability (3 and Δ)

$$P(3 \text{ and } \Delta) = \frac{4}{28} \times \frac{6}{28}$$
$$= \frac{24}{784}$$
$$= \frac{3}{98}$$

(5) 16. Recall that a **simulation** is a method used to answer questions about real-life situations. Using the random number table, design a model to simulate the following situation. Be sure to detail your simulation, explaining how you plan to use the given table.

299653	968783	815850	176081	413855	398427	865891	209529
164175	399313	418329	216183	632158	447330	994581	541547
998071	614526	504546	596562	338483	661860	424844	592343
130489	351868	436835	754219	993825	475008	605644	658816
702124	483426	581458	437943	494559	325889	604332	193016
619128	577649	993767	263786	238837	358967	404299	495113
632010	636070	328356	245530	718002	698271	827630	881365

Situation: Marvin, a student, determines that he does not have time to study for an exam. The exam only consists of 6 multiple choice questions with answer options a., b., or c. He hopes that he can guess at least 4 out of the 6 answers correctly. Estimate the probability that he will actually be able to do this. You may wish to use a table of this form.

Number of Favorable Events	Number of Trials	Estimated Probability

Solutions will vary.

A digit is selected in the body of the table. After each digit has been used, the digit to its right will be used next. If a digit is 0 to 2, a correct answer is assumed. If a digit is 3 to 8, an incorrect answer is assumed. The digit 9 is not used. This will be done for 100 sets of 6 questions.

Number of Favorable Events	Number of Trials	Estimated Probability
0	6	0.06
1	23	0.23
2	39	0.3922
3	22	0.227
4	7	0.07
5	3	0.03
6	0	0

$$P(4 \text{ or more}) \doteq 0.07 + 0.03 + 0$$
$$\doteq 0.10$$

The probability that Marvin will get 4 or more answers correct is approximately 0.10.

(5) 17. Loretta decides that she will simply guess at the answers to a 10-question true/false test. Before she writes the exam she simulates the situation using a coin and by letting **heads** represent a correct response. The following table is a summary of the outcomes of her simulation for the number of heads occurring per trial of 10 coin tosses.

Number of Heads	Number of Trials	Estimated Probability
0	2	$\frac{1}{72}$
1	3	$\frac{1}{48}$
2	9	$\frac{1}{16}$
3	17	$\frac{17}{144}$
4	28	$\frac{7}{36}$
5	34	$\frac{17}{72}$
6	26	$\frac{13}{72}$
7	13	$\frac{13}{144}$
8	7	$\frac{7}{144}$
9	5	$\frac{5}{144}$
10	0	0

Complete the above table and determine the probability that she will obtain the following scores. Express your answers as reduced fractions.

a. not less than 60%

$$P(\not< 60\%) = \frac{13}{72} + \frac{13}{144} + \frac{7}{144} + \frac{5}{144} + 0$$
$$= \frac{26+13+7+5+0}{144}$$
$$= \frac{51}{144}$$
$$= \frac{17}{48}$$

b. between 30% and 70% (inclusive)

$$P(30\% \le s \le 70\%) = \frac{17}{144} + \frac{7}{36} + \frac{17}{72} + \frac{13}{72} + \frac{13}{144}$$
$$= \frac{17+28+34+26+13}{144}$$
$$= \frac{118}{144}$$
$$= \frac{59}{72}$$

(5) 18. Circle the correct response.
Determine the most appropriate simulation for the following situation.
Mary has two pairs of shoes to choose from to wear to school. She randomly selects a pair of shoes each morning. What is the probability that she wears the same pair of shoes for a week?

A. Use a random number table considering odd and even numbers only.

B. Draw 2 numbers from a hat.

C. Toss a coin.

(D.) All of the above.

(5) 19. Circle the correct response.
Determine the most appropriate simulation for the following situation.
Mary has five pairs of shoes to choose from to wear to school. She randomly selects a pair of shoes each morning. What is the probability that she wears the same pair of shoes for six consecutive days?

A. Use a random number table of 6-digit numbers.

B. Roll a die.

C. Toss a coin 3 times.

(D.) All of the above.

(5) 20. In many countries, it is the son who takes care of the parents when they have reached their old age. Because of this, couples are inclined to continue to have children until they have a son.

a. Using this situation as the rule, do a simulation to find the following information.

i. the average number of children per family

Solutions will vary. This is one possible simulation.

B	B	GGGB	GB	B
GGGGB	GGB	GGB	B	GGB
B	GGGB	B	B	B
B	B	GGB	B	B
B	B	GB	GGGB	GB
GGB	GB	GGB	GB	GB
B	B	GGB	B	GB
GGGGGGGB	GGB	GGGB	GB	B
B	GB	B	GB	GGGB
GGB	B	B	GGGGB	GB

$\frac{109}{50}$ or about 2

There will be an average of 2 children per family.

ii. the average number of girls per family

Solutions will vary.

$\frac{59}{50}$ or about 1

There will be an average of 1 girl per family.

b. Would you expect more girls or more boys to be born? Explain your answer.

The answers will vary. Check to see how they support their answer.

Topic 1

_____ marks

Final Test

There are two copies of the final test: the student's copy which is perforated and designed for photocopying and possible faxing and the teacher's copy which includes a marking guide.

Note:

The student's copy of this final test should be kept secure by the teacher. Students should not have access to this test until it is assigned in a supervised situation. The answers should be stored securely and retained by the teacher at all times.

MATHEMATICS 23

FINAL TEST

GENERAL INSTRUCTIONS

1. The time limit for this test is **2 hours.**

2. There are **150 marks** on this exam.

   PART A: Multiple Choice - 30 marks

   PART B: Written Response - 120 marks

3. Calculators that are not programmed may be used.

4. Trigonometric tables are provided at the end of the test.

5. It is better to complete all the work you know well first. After that you can deal with the difficult ones. Do not spend too much time on any one problem. If you finish the test before the two-hour time limit, go back and check your answers.

PART A: MULTIPLE CHOICE

All multiple-choice questions must be answered on the Part A Response Page included in your test.

Read carefully and decide which of the choices BEST completes the statement or answers the question. Locate the question number on the Response Page and place the appropriate letter A, B, C, D, or E in the blank.

Example:

83. The value of -5^2 is

A. 10

B. -10

C. 25

D. -25

E. none of the above

Response Page

__D__ 83.

1. The value of $\dfrac{(-27)^{\frac{4}{3}}}{(64)^{-\frac{1}{3}}}$ is

A. $\dfrac{16}{81}$

B. $\dfrac{81}{4}$

C. 81

D. 324

E. none of the above

2. The G.C.F. of $15x^3y^2z$, $30x^2y^2z^3$, and $60x^4y^3z^2$ is

   A. $60x^4y^3z^3$

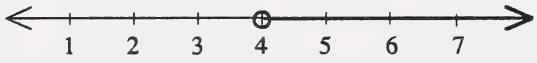
   B. $15x^2y^2z$

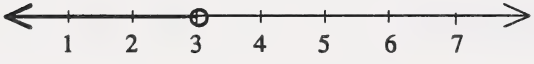
   C. $30x^4y^3z^3$

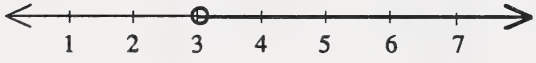
   D. $60x^2y^2z$

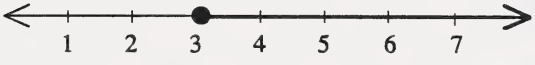
   E. none of the above

3. The graph of the solution to the inequality $-5x+7<-8$ is

   A. 

   B. 

   C. 

   D. 

   E. none of the above

4. The straight-line graph of $y = -4$ has the characteristic that it

   A. is parallel to the y-axis

   B. has an x-intercept

   C. has no y-intercept

   D. has a slope of 0

   E. none of the above

5. The slope of a line perpendicular to the graph of $y = 7x + 3$ is

   A. 7

   B. -7

   C. $\frac{1}{7}$

   D. $-\frac{1}{7}$

   E. none of the above

6. A linear equation system in which the slopes of the two equations are the same and the y-intercepts are the same has

   A. no solution

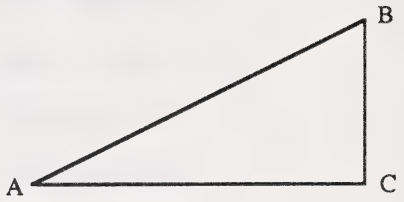
   B. one solution

   C. two solutions

   D. an infinite number of solutions

   E. none of the above

7. With respect to $\angle A$, the trigonometric ratio $\frac{BC}{AC}$ is called the



A. tangent of $\angle A$

B. cosine of $\angle A$

C. sine of $\angle A$

D. secant of $\angle A$

E. none of the above

8. The formula $A = 4\pi r^2$ finds the surface area of a

A. cone

B. cylinder

C. dome

D. sphere

E. none of the above

9. If $\sin\theta = \frac{\sqrt{3}}{2}$, what is $\cos\theta$?

A. 2

B. $\frac{1}{2}$

C. $\sqrt{3}$

D. $\frac{2}{\sqrt{3}}$

E. none of the above

10. The probability of rolling a 2, 3, or 4 using a die is

A. $\frac{3}{5}$

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

E. none of the above

11. The value of $4^{-2}\left(\frac{1}{5}\right)^{-2}(-2)^{-1}$ is

A. $\frac{25}{32}$

B. $-\frac{25}{32}$

C. $\frac{32}{25}$

D. $-\frac{32}{25}$

E. none of the above

12. The amount paid on a $9200 loan at the end of 5 years at 8% per annum compounded semiannually is approximately

A. $12 967.16

B. $13 053.32

C. $13 412.76

D. $13 618.25

E. none of the above

13. Suppose you toss a coin and record whether it lands heads (H) or tails (T). If you toss the coin 3 times, the probability of recording HTH is

A. $\frac{1}{9}$

B. $\frac{1}{8}$

C. $\frac{1}{6}$

D. $\frac{1}{3}$

E. none of the above

14. Determine which statement is NOT true.

A. The area of a trapezoid is $\frac{1}{2}h(a+b)$.

B. The volume of a sphere with a radius of 3 cm is $113\ \text{cm}^3$.

C. A pyramid is a solid that has a polygon as a base.

D. Euler's formula is $V + E - F = 2$.

E. none of the above

15. The solution to the linear system $5x - 2y = -19$ and $3x + 3y = -3$ is

A. $(3, -2)$

B. $(3, 2)$

C. $(-3, 2)$

D. $(-3, -2)$

E. none of the above

PART A: RESPONSE PAGE

D	1.	B	9.
B	2.	C	10.
C	3.	B	11.
D	4.	D	12.
D	5.	B	13.
D	6.	D	14.
A	7.	C	15.
D	8.		

Value

PART B: WRITTEN RESPONSE

Be sure to show all your work. Part marks will be given for partially correct solutions.

1. Simplify each of the following.

(2) a. $(10^4)^2 \times 6^0$

$= 10^8 \times 1$

$= 10^8$

(2) b. $\dfrac{10}{6^{-2}}$

$= 10 \times 6^2$

$= 10 \times 36$

$= 360$

(3) c. $\sqrt[3]{\dfrac{250}{16}}$

$= \sqrt[3]{\dfrac{125}{8}}$

$= \dfrac{5}{2}$

(2) d. $343^{\frac{2}{3}}$

$= 7^2$

$= 49$

(2) e. $\left(\dfrac{x^2y^2}{d^4} \times \dfrac{d^3xy^3}{y^4}\right)^2$

$= \left(\dfrac{x^3y^5}{dy^4}\right)^2$

$= \left(\dfrac{x^3y}{d}\right)^2$

$= \dfrac{x^6y^2}{d^2}$

Value

2. Factor fully.

(1) a. $8x^2y^3-24x^2y^5$

$=8x^2y^3(1-3y^2)$

(2) b. $2x(x-6y)-7(x-6y)$

$=(2x-7)(x-6y)$

(2) c. $(x+y)^2-16$

$=(x+y+4)(x+y-4)$

(2) d. $8x^2+10x-3$

$=(4x-1)(2x+3)$

(2) 3. Solve $8-3x\le -7$.

$8-3x\le -7$

$-3x\le -7-8$

$-3x\le -15$

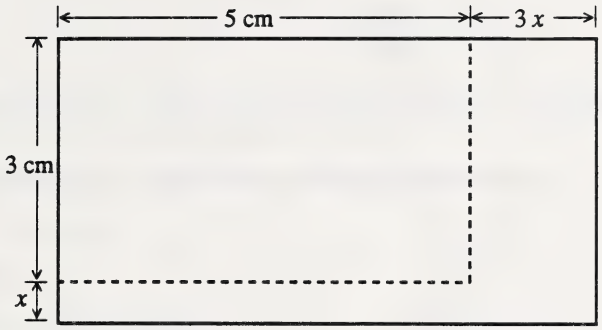
$x\ge 5$

TEACHER'S COPY

Value

(5)

4. A rectangular piece of metal is increased by x cm in one dimension and $3x$ cm in the other dimension. Determine the new dimensions of the metal if the new plate has an area of $84\ \text{cm}^2$.



$$(3+x)(5+3x)=84$$
$$15+5x+9x+3x^2=84$$
$$3x^2+14x-69=0$$
$$(3x+23)(x-3)=0$$

$3x+23=0$ $\qquad$ $x-3=0$

$x=-\frac{23}{3}$ $\qquad$ $x=3$

The dimensions are $(3+x)$ by $(5+3x)$
$(3+3)$ by $[(5+3(3)]$
6 by 14

The dimensions are 6 cm × 14 cm.

TEACHER'S COPY

Value

(4) 5. How much money must be invested today at 9% per annum compounded semiannually in order to produce $6800 in 7 years?

$$A = P(1+i)^n$$
$$6800 = P(1.045)^{14}$$
$$P = \frac{6800}{(1.045)^{14}}$$
$$P = \$3671.82$$

(2) 6. Determine the slope of the line through A(– 6, 5) and B(– 2, 9).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{9-5}{-2-(-6)}$$
$$= \frac{4}{-2+6}$$
$$= \frac{4}{4}$$
$$= 1$$

(3) 7. Write the equation $7x - 3y - 6 = 0$ in the form $y = mx + b$ and state the slope and the y-intercept.

$$7x - 3y - 6 = 0$$
$$-3y = -7x + 6$$
$$y = \frac{-7x}{-3} + \frac{6}{-3}$$
$$y = \frac{7}{3}x - 2$$

slope = $\frac{7}{3}$

y-intercept = -2

TEACHER'S COPY

Value

(4) 8. Determine the equation of the line that is parallel to $x = -3$ and passes through the point $(4, -8)$.

$x = 4$

(5) 9. Write the equation of the line that is perpendicular to $y = -5x + 7$ and passes through the point $(2, -8)$. Write the equation in the general form $ax + by + c = 0$.

$$y = -5x + 7$$

$$\text{slope} = -5$$

The slope of the required line is $\frac{1}{5}$.

$$\frac{y-(-8)}{x-2} = \frac{1}{5}$$

$$\frac{y+8}{x-2} = \frac{1}{5}$$

$$5(y+8) = 1(x-2)$$

$$5y + 40 = x - 2$$

$$x - 5y - 42 = 0$$

Value

(3) 10. The sum of two numbers is 32. Two times the smaller number is 10 more than the larger number. Determine the two numbers.

$$\begin{aligned} x+y &= 32 \quad (1) \\ 2x-y &= 10 \quad (2) \\ \hline 3x &= 42 \quad (1)+(2) \\ x &= 14 \end{aligned}$$

$$\begin{aligned} x+y &= 32 \quad (1) \\ 14+y &= 32 \\ y &= 18 \end{aligned}$$

(5) 11. A rectangular field has a perimeter of 4080 m. The length is 240 m less than twice the width. Determine the length and width of the field. Use a final statement in your answer.

Let l = length
and w = width.

$$\begin{aligned} 2w+2l &= 4080 \quad (1) \\ 2w-l &= 240 \quad (2) \\ \hline 3l &= 3840 \quad (1)-(2) \\ l &= 1280 \\ 2w-1280 &= 240 \quad (2) \\ 2w &= 1280+240 \\ 2w &= 1520 \\ w &= 760 \end{aligned}$$

The width is 760 m and the length is 1280 m.

TEACHER'S COPY

Value

(6)

12. Solve the system.

$$6x-4y=3$$
$$9x+8y=1$$

$$\begin{aligned} 6x-4y&=3 && (1)\times 2\\ 9x+8y&=1 && (2)\end{aligned}$$

$$\begin{aligned} 12x-8y&=6 && (3)\\ 9x+8y&=1 && (2)\\ \hline 21x&=7 && (3)+(2)\end{aligned}$$

$$x=\frac{7}{21}$$

$$x=\frac{1}{3}$$

$$\begin{aligned} 6\left(\frac{1}{3}\right)-4y&=3 && (1)\\ 2-4y&=3\\ -4y&=3-2\\ -4y&=1\\ y&=-\frac{1}{4}\end{aligned}$$

$$x=\frac{1}{3},\ y=-\frac{1}{4}$$

Value

(6)

13. Four beefburgers and three glasses of milk cost \$8.50. Two beefburgers and one glass of milk cost \$3.90. Determine the cost of one beefburger and the cost of one glass of milk.

Let x = cost of one beefburger.
Let y = cost of one glass of milk.

$$\begin{aligned} 4x+3y&=8.50 && (1)\\ 2x+1y&=3.90 && (2)\times -2 \end{aligned}$$

$$\begin{aligned} 4x+3y&=8.50 && (1)\\ -4x-2y&=-7.80 && (3)\\ \hline y&=0.70 && (1)+(3) \end{aligned}$$

$$\begin{aligned} 2x+1y&=3.90 && (2)\\ 2x+0.70&=3.90\\ 2x&=3.20\\ x&=1.60 \end{aligned}$$

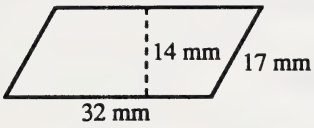
One beefburger costs \$1.60 and one glass of milk costs 70¢.

TEACHER'S COPY

Value

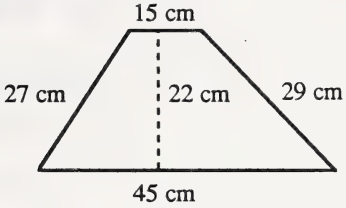
(10) 14. Calculate the following areas.

a.



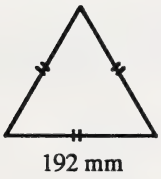
$$\text{Area} = \text{base} \times \text{height}$$
$$= 32 \text{ mm} \times 14 \text{ mm}$$
$$= 448 \text{ mm}^2$$

b.



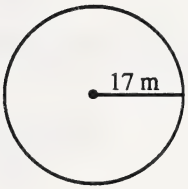
$$\text{Area} = \frac{1}{2} \times \text{height}(\text{base} + \text{base})$$
$$= \frac{1}{2} \times 22 \text{ cm}(15 \text{ cm} + 45 \text{ cm})$$
$$= \frac{1}{2} \times 22 \text{ cm}(60 \text{ cm})$$
$$= 660 \text{ cm}^2$$

c.



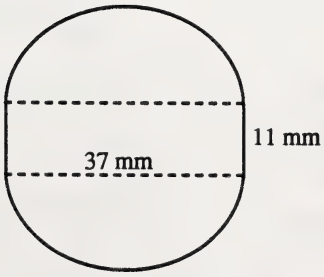
$$\text{Area} = \frac{\sqrt{3}}{4} \times \text{side}^2$$
$$= \frac{\sqrt{3}}{4} \times (192 \text{ mm})^2$$
$$\doteq 15\,963 \text{ mm}^2$$

d.



$$\text{Area} = \pi \times \text{radius}^2$$
$$= \pi(17 \text{ m})^2$$
$$= \pi \times 289 \text{ m}^2$$
$$\doteq 907.9 \text{ m}^2$$

e.



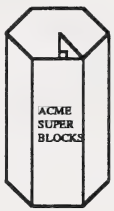
$$\text{Area} = \text{Area circle} + \text{Area rectangle}$$
$$= \pi \times \text{radius}^2 + \text{length} \times \text{width}$$
$$= \pi\left(\frac{37 \text{ mm}}{2}\right)^2 + 37 \text{ mm} \times 11 \text{ mm}$$
$$= \pi \times 342.25 \text{ mm}^2 + 407 \text{ mm}^2$$
$$\doteq 1075 \text{ mm}^2 + 407 \text{ mm}^2$$
$$\doteq 1482 \text{ mm}^2$$

TEACHER'S COPY

Value

(5) 15. What volume will be displaced by a regular-hexagonal prism that is used for packaging toy building blocks? Each side of the hexagon is 8 cm. The radius of the hexagon is 7 cm. The height of the container is 22 cm.

$$\text{Volume} = \text{Area of base} \times \text{height}$$
$$= 6 \times \text{radius} \times \frac{1}{2}\text{base} \times \text{height}$$
$$= 6 \times 7 \text{ cm} \times 4 \text{ cm} \times 22 \text{ cm}$$
$$= 3696 \text{ cm}^3$$



The prism will displace 3696 cm^3.

(5) 16. What is the diameter of the moon Europa if it has a surface area of 30 270 000 km^2?

$$\text{Surface Area} = 4\pi \times \text{radius}^2$$
$$30\,270\,000 \text{ km}^2 = 4\pi \times \text{radius}^2$$

$$\frac{30\,270\,000 \text{ km}^2}{4\pi} = \text{radius}^2$$

$$\sqrt{\frac{30\,270\,000 \text{ km}^2}{4\pi}} = \text{radius}^2$$

$$\sqrt{2\,408\,810 \text{ km}^2} \doteq \text{radius}$$

$$1552 \text{ km} \doteq \text{radius}$$

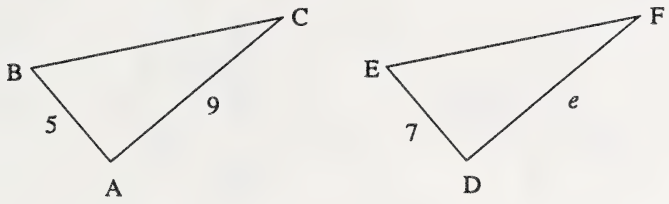
$$\text{diameter} = 2 \times \text{radius}$$
$$\text{diameter} = 2 \times 1552 \text{ km}$$
$$\text{diameter} = 3104 \text{ km}$$

Europa has a diameter of 3104 km.

TEACHER'S COPY

Value

(4) 17. Given $\Delta ABC \sim \Delta DEF$, determine the value of e to one decimal place.



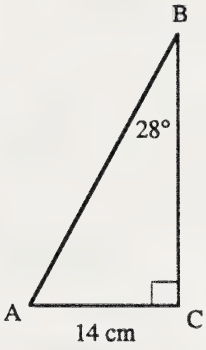
$$\frac{5}{9} = \frac{7}{e}$$

$$5e = 9(7)$$

$$e = \frac{9(7)}{5}$$

$$e = 12.6$$

(5) 18. Given a right triangle ABC, where $\angle B = 28°$ and side $b = 14$ cm, determine the length of the hypotenuse of the triangle. Round the final answer to two decimal places.



$$\sin 28° = \frac{14}{AB}$$

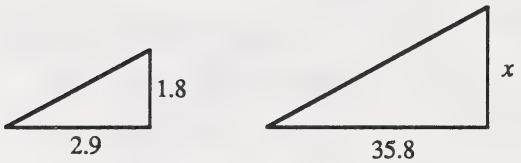
$$AB = \frac{14}{\sin 28°}$$

$$= \frac{14}{0.469\ 47}$$

$$AB \doteq 29.82 \text{ cm}$$

Value

(5) 19. A post 1.8 m tall casts a shadow 2.9 m along the ground. At the same time, a building has a shadow 35.8 m long. Determine the height of the building to one decimal place.



$$\frac{1.8}{2.9} = \frac{x}{35.8}$$

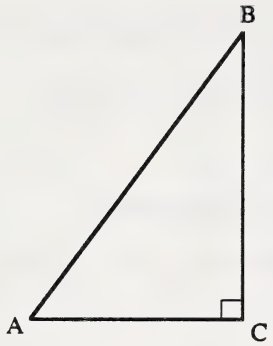
$$x = \frac{1.8(35.8)}{2.9}$$

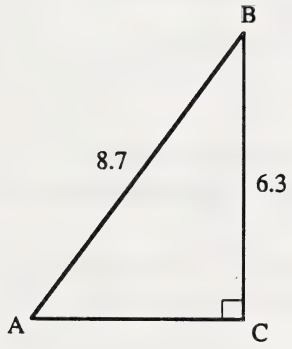
$$x = 22.2 \text{ m}$$

TEACHER'S COPY

Value

(6) 20. Solve ΔABC if $\angle C = 90°$, BA = 8.7 cm, and BC = 6.3 cm. Round the angle measurements to a whole number and the side measurement to one decimal place.





$$\sin A = \frac{6.3}{8.7}$$
$$= 0.7241$$
$$\angle A = 46°$$
$$\angle B = 90° - 46°$$
$$= 44°$$

$$\cos A = \frac{AC}{AB}$$
$$\cos A = \frac{AC}{8.7}$$
$$AC = 8.7 \cos A$$
$$= 8.7 \cos 46°$$
$$AC \doteq 6.0 \text{ cm}$$

TEACHER'S COPY

Value

(5) 21. There are 15 cards of equal size. Each card has a number from 1 to 15. The cards are placed in a hat and mixed. Determine the theoretical probabilities for the following situations.

a. What is the probability of drawing a number greater than 11?

The possible numbers are 12, 13, 14, 15. $P(>11)=\frac{4}{15}$

b. What is the probability of drawing an odd number?

The possible numbers are 1, 3, 5, 7, 9, 11, 13, 15. $P(\text{odd})=\frac{8}{15}$

c. What is the probability of drawing a prime number?

The possible numbers are 2, 3, 4, 7, 11, 13. $P(\text{prime})=\frac{6}{15}$

$=\frac{2}{5}$

d. What is the probability of drawing a number greater than 6 but less than 12?

The possible numbers are 7, 8, 9, 10, 11. $P(6<x<12)=\frac{5}{15}$

$=\frac{1}{3}$

e. What is the probability of drawing a number less than 6 and greater than 10?

This is an impossible case. $P(10<x<6)=\frac{0}{15}$

$=0$

Value

22. Use the information in the table to answer the following questions.

Math Marks	
Mark (%)	Frequency
0 - 40	4
41 - 45	6
46 - 50	5
51 - 55	7
56 - 60	8
61 - 65	12
66 - 70	13
71 - 75	12
76 - 80	9
81 - 85	8
86 - 90	2
91 - 95	2
96 - 100	1
Total	89

(1) a. What is the probability that a mark was above 90%?

$$P(>90\%) = \frac{3}{89}$$
$$\doteq 0.033$$

(2) b. What is the probability that a mark was from 51 to 75?

$$P(51 \text{ to } 75) = \frac{52}{89}$$
$$\doteq 0.584$$

(2) c. A mark over 50 but less than 66 is a C standing. What percent of the students receive C's?

$$P(50 \text{ to } 66) = \frac{27}{89}$$
$$\doteq 0.303$$

(2) d. If 118 students wrote the exam, determine how many students would receive a mark from 66 to 80%.

$$P(66 \text{ to } 80) = \frac{34}{89}$$
$$\doteq 0.382$$
$$0.382 \times 118 = 45.07$$
$$\doteq 45$$

TEACHER'S COPY

Value

23. An office worker has ten different hats. This worker decides to randomly select a hat to wear to the office each day for the five-day work week. Use a simulation to answer the following questions. Use the random number table of 50 trials given below and let the digits 0 to 9 each represent a different hat. Each block of 5 will represent a five-day week. To answer the questions, each trial must be checked.

52097	96239	72255	86588	13395	60424	63156	92309	79545	64637
58233	53891	61336	38086	39495	37774	27949	34573	58358	54006
10884	42540	17696	86020	78016	38734	04767	49518	19256	24990
35593	83183	91244	68292	24406	06991	47542	18392	25866	76750
67050	00008	52103	59222	26126	74210	01129	15483	86292	53468

(3) a. Estimate the probability that this worker will wear a different hat each day.

There are 10 blocks that have five different numbers.

approximately $\frac{10}{50}$

(1) b. Estimate the probability that this worker will wear the same hat each day.

There is no trial where a digit repeats 5 times.

approximately 0

(1) c. Estimate the probability that this worker will wear the same hat more than once in one week.

approximately $1-\frac{10}{50}=\frac{40}{50}=\frac{4}{5}$

TABLE OF TRIGONOMETRIC FUNCTIONS

ANGLE deg	sin	cos	tan	csc	sec	cot
0	0.0000	1.0000	0.0000	undefined	1.0000	undefined
1	0.0175	0.9998	0.0175	57.299	1.0002	57.2900
2	0.0349	0.9994	0.0349	28.654	1.0006	28.6363
3	0.0523	0.9986	0.0524	19.107	1.0014	19.0811
4	0.0698	0.9976	0.0699	14.336	1.0024	14.3007
5	0.0872	0.9962	0.0875	11.474	1.0038	11.4301
6	0.1045	0.9945	0.1051	9.5668	1.0055	9.5144
7	0.1219	0.9925	0.1228	8.2055	1.0075	8.1443
8	0.1392	0.9903	0.1405	7.1853	1.0098	7.1154
9	0.1564	0.9877	0.1584	6.3925	1.0125	6.3138
10	0.1736	0.9848	0.1763	5.7588	1.0154	5.6713
11	0.1908	0.9816	0.1944	5.2408	1.0187	5.1446
12	0.2079	0.9781	0.2126	4.8097	1.0223	4.7046
13	0.2250	0.9744	0.2309	4.4454	1.0263	4.3315
14	0.2419	0.9703	0.2493	4.1336	1.0306	4.0108
15	0.2588	0.9659	0.2679	3.8637	1.0353	3.7321
16	0.2756	0.9613	0.2867	3.6280	1.0403	3.4874
17	0.2924	0.9563	0.3057	3.4208	1.0457	3.2709
18	0.3090	0.9511	0.3249	3.2361	1.0515	3.0777
19	0.3256	0.9455	0.3443	3.0716	1.0576	2.9042
20	0.3420	0.9397	0.3640	2.9238	1.0642	2.7475
21	0.3584	0.9336	0.3839	2.7904	1.0711	2.6051
22	0.3746	0.9272	0.4040	2.6695	1.0785	2.4751
23	0.3907	0.9205	0.4245	2.5593	1.0864	2.3559
24	0.4067	0.9135	0.4452	2.4586	1.0946	2.2460
25	0.4226	0.9063	0.4663	2.3662	1.1034	2.1445
26	0.4384	0.8988	0.4877	2.2812	1.1126	2.0503
27	0.4540	0.8910	0.5095	2.2027	1.1223	1.9626
28	0.4695	0.8829	0.5317	2.1301	1.1326	1.8807
29	0.4848	0.8746	0.5543	2.0627	1.1434	1.8040
30	0.5000	0.8660	0.5774	2.0000	1.1547	1.7321
31	0.5150	0.8572	0.6009	1.9416	1.1667	1.6643
32	0.5299	0.8480	0.6249	1.8871	1.1792	1.6003
33	0.5446	0.8387	0.6494	1.8361	1.1924	1.5399
34	0.5592	0.8290	0.6745	1.7883	1.2062	1.4826
35	0.5736	0.8192	0.7002	1.7435	1.2208	1.4281
36	0.5878	0.8090	0.7265	1.7013	1.2361	1.3764
37	0.6018	0.7986	0.7536	1.6616	1.2521	1.3270
38	0.6157	0.7880	0.7813	1.6243	1.2690	1.2799
39	0.6293	0.7771	0.8098	1.5890	1.2868	1.2349
40	0.6428	0.7660	0.8391	1.5557	1.3054	1.1918
41	0.6561	0.7547	0.8693	1.5243	1.3250	1.1504
42	0.6691	0.7431	0.9004	1.4945	1.3456	1.1106
43	0.6820	0.7314	0.9325	1.4663	1.3673	1.0724
44	0.6947	0.7193	0.9657	1.4396	1.3902	1.0355
45	0.7071	0.7071	1.0000	1.4142	1.4142	1.0000

ANGLE deg	sin	cos	tan	csc	sec	cot
45	0.7071	0.7071	1.0000	1.4142	1.4142	1.0000
46	0.7193	0.6947	1.0355	1.3901	1.4396	0.9657
47	0.7314	0.6820	1.0724	1.3673	1.4663	0.9325
48	0.7431	0.6691	1.1106	1.3456	1.4945	0.9004
49	0.7547	0.6561	1.1504	1.3250	1.5243	0.8693
50	0.7660	0.6428	1.1918	1.3054	1.5557	0.8391
51	0.7771	0.6293	1.2349	1.2868	1.5890	0.8098
52	0.7880	0.6157	1.2799	1.2690	1.6243	0.7813
53	0.7986	0.6018	1.3270	1.2521	1.6616	0.7536
54	0.8090	0.5878	1.3764	1.2361	1.7013	0.7265
55	0.8192	0.5736	1.4281	1.2208	1.7435	0.7002
56	0.8290	0.5592	1.4826	1.2062	1.7883	0.6745
57	0.8387	0.5446	1.5399	1.1924	1.8361	0.6494
58	0.8480	0.5299	1.6003	1.1792	1.8871	0.6249
59	0.8572	0.5150	1.6643	1.1666	1.9416	0.6009
60	0.8660	0.5000	1.7321	1.1547	2.0000	0.5774
61	0.8746	0.4848	1.8040	1.1434	2.0627	0.5543
62	0.8829	0.4695	1.8807	1.1326	2.1301	0.5317
63	0.8910	0.4540	1.9626	1.1223	2.2027	0.5095
64	0.8988	0.4384	2.0503	1.1126	2.2812	0.4877
65	0.9063	0.4226	2.1445	1.1034	2.3662	0.4663
66	0.9135	0.4067	2.2460	1.0946	2.4586	0.4452
67	0.9205	0.3907	2.3559	1.0864	2.5593	0.4245
68	0.9272	0.3746	2.4751	1.0785	2.6695	0.4040
69	0.9336	0.3584	2.6051	1.0712	2.7904	0.3839
70	0.9397	0.3420	2.7475	1.0642	2.9238	0.3640
71	0.9455	0.3256	2.9042	1.0576	3.0716	0.3443
72	0.9511	0.3090	3.0777	1.0515	3.2361	0.3249
73	0.9563	0.2924	3.2709	1.0457	3.4203	0.3057
74	0.9613	0.2756	3.4874	1.0403	3.6280	0.2867
75	0.9659	0.2588	3.7321	1.0353	3.8637	0.2679
76	0.9703	0.2419	4.0108	1.0306	4.1336	0.2493
77	0.9744	0.2250	4.3315	1.0263	4.4454	0.2309
78	0.9781	0.2079	4.7046	1.0223	4.8097	0.2126
79	0.9816	0.1908	5.1446	1.0187	5.2408	0.1944
80	0.9848	0.1736	5.6713	1.0154	5.7588	0.1763
81	0.9877	0.1564	6.3138	1.0125	6.3925	0.1584
82	0.9903	0.1392	7.1154	1.0098	7.1853	0.1405
83	0.9925	0.1219	8.1443	1.0075	8.2055	0.1228
84	0.9945	0.1045	9.5144	1.0055	9.5668	0.1051
85	0.9962	0.0872	11.4301	1.0038	11.474	0.0875
86	0.9976	0.0698	14.3007	1.0024	14.336	0.0699
87	0.9986	0.0523	19.0811	1.0014	19.107	0.0524
88	0.9994	0.0349	28.6363	1.0006	28.654	0.0349
89	0.9998	0.0175	57.2900	1.0002	57.299	0.0175
90	1.0000	0.0000	undefined	1.0000	undefined	0.0000

MATHEMATICS 23

FINAL TEST

GENERAL INSTRUCTIONS

1. The time limit for this test is **2 hours.**

2. There are **150 marks** on this exam.

   PART A: Multiple Choice - 30 marks

   PART B: Written Response - 120 marks

3. Calculators that are not programmed may be used.

4. Trigonometric tables are provided at the end of the test.

5. It is better to complete all the work you know well first. After that you can deal with the difficult ones. Do not spend too much time on any one problem. If you finish the test before the two-hour time limit, go back and check your answers.

PART A: MULTIPLE CHOICE

All multiple-choice questions must be answered on the Part A Response Page included in your test.

Read carefully and decide which of the choices BEST completes the statement or answers the question. Locate the question number on the Response Page and place the appropriate letter A, B, C, D, or E in the blank.

Example:

83. The value of -5^2 is

A. 10

B. -10

C. 25

D. -25

E. none of the above

Response Page

__D__ 83.

1. The value of $\dfrac{(-27)^{\frac{4}{3}}}{(64)^{-\frac{1}{3}}}$ is

A. $\dfrac{16}{81}$

B. $\dfrac{81}{4}$

C. 81

D. 324

E. none of the above

2. The G.C.F. of $15x^3y^2z$, $30x^2y^2z^3$, and $60x^4y^3z^2$ is

   A. $60x^4y^3z^3$

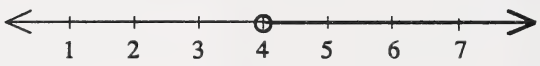
   B. $15x^2y^2z$

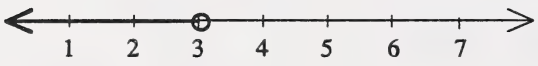
   C. $30x^4y^3z^3$

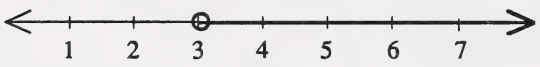
   D. $60x^2y^2z$

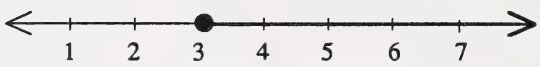
   E. none of the above

3. The graph of the solution to the inequality $-5x+7<-8$ is

   A. 

   B. 

   C. 

   D. 

   E. none of the above

4. The straight-line graph of $y = -4$ has the characteristic that it

   A. is parallel to the y-axis

   B. has an x-intercept

   C. has no y-intercept

   D. has a slope of 0

   E. none of the above

5. The slope of a line perpendicular to the graph of $y = 7x + 3$ is

   A. 7

   B. -7

   C. $\frac{1}{7}$

   D. $-\frac{1}{7}$

   E. none of the above

6. A linear equation system in which the slopes of the two equations are the same and the y-intercepts are the same has

   A. no solution

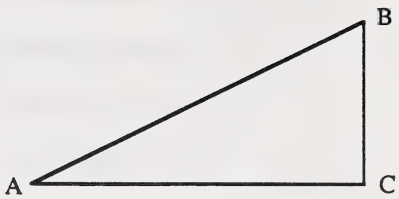
   B. one solution

   C. two solutions

   D. an infinite number of solutions

   E. none of the above

7. With respect to $\angle A$, the trigonometric ratio $\frac{BC}{AC}$ is called the



A. tangent of $\angle A$

B. cosine of $\angle A$

C. sine of $\angle A$

D. secant of $\angle A$

E. none of the above

8. The formula $A = 4\pi r^2$ finds the surface area of a

A. cone

B. cylinder

C. dome

D. sphere

E. none of the above

9. If $\sin\theta = \frac{\sqrt{3}}{2}$, what is $\cos\theta$?

A. 2

B. $\frac{1}{2}$

C. $\sqrt{3}$

D. $\frac{2}{\sqrt{3}}$

E. none of the above

10. The probability of rolling a 2, 3, or 4 using a die is

A. $\frac{3}{5}$

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

E. none of the above

11. The value of $4^{-2}\left(\frac{1}{5}\right)^{-2}(-2)^{-1}$ is

A. $\frac{25}{32}$

B. $-\frac{25}{32}$

C. $\frac{32}{25}$

D. $-\frac{32}{25}$

E. none of the above

12. The amount paid on a $9200 loan at the end of 5 years at 8% per annum compounded semiannually is approximately

A. $12 967.16

B. $13 053.32

C. $13 412.76

D. $13 618.25

E. none of the above

13. Suppose you toss a coin and record whether it lands heads (H) or tails (T). If you toss the coin 3 times, the probability of recording HTH is

A. $\frac{1}{9}$

B. $\frac{1}{8}$

C. $\frac{1}{6}$

D. $\frac{1}{3}$

E. none of the above

14. Determine which statement is NOT true.

A. The area of a trapezoid is $\frac{1}{2}h(a+b)$.

B. The volume of a sphere with a radius of 3 cm is $113\ \text{cm}^3$.

C. A pyramid is a solid that has a polygon as a base.

D. Euler's formula is $V + E - F = 2$.

E. none of the above

15. The solution to the linear system $5x - 2y = -19$ and $3x + 3y = -3$ is

A. $(3, -2)$

B. $(3, 2)$

C. $(-3, 2)$

D. $(-3, -2)$

E. none of the above

PART A: RESPONSE PAGE

_____	1.	_____	9.
_____	2.	_____	10.
_____	3.	_____	11.
_____	4.	_____	12.
_____	5.	_____	13.
_____	6.	_____	14.
_____	7.	_____	15.
_____	8.		

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

PART B: WRITTEN RESPONSE

Be sure to show all your work. Part marks will be given for partially correct solutions.

1. Simplify each of the following.

(2) a. $\left(10^4\right)^2 \times 6^0$

(2) b. $\dfrac{10}{6^{-2}}$

(3) c. $\sqrt[3]{\dfrac{250}{16}}$

(2) d. $343^{\frac{2}{3}}$

(2) e. $\left(\dfrac{x^2y^2}{d^4} \times \dfrac{d^3xy^3}{y^4}\right)^2$

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

2. Factor fully.

(1) a. $8x^2y^3-24x^2y^5$

(2) b. $2x(x-6y)-7(x-6y)$

(2) c. $(x+y)^2-16$

(2) d. $8x^2+10x-3$

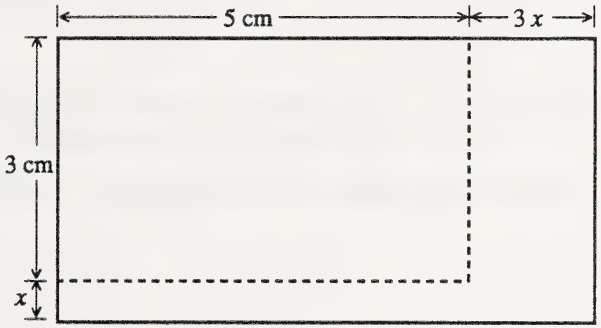
(2) 3. Solve $8-3x \leq -7$.

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

(5) 4. A rectangular piece of metal is increased by x cm in one dimension and $3x$ cm in the other dimension. Determine the new dimensions of the metal if the new plate has an area of $84\ cm^2$.



Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

(4) 5. How much money must be invested today at 9% per annum compounded semiannually in order to produce $6800 in 7 years?

(2) 6. Determine the slope of the line through A(– 6, 5) and B(– 2, 9).

(3) 7. Write the equation $7x - 3y - 6 = 0$ in the form $y = mx + b$ and state the slope and the y-intercept.

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

(4) 8. Determine the equation of the line that is parallel to $x = -3$ and passes through the point $(4, -8)$.

(5) 9. Write the equation of the line that is perpendicular to $y = -5x + 7$ and passes through the point $(2, -8)$. Write the equation in the general form $ax + by + c = 0$.

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

(3) 10. The sum of two numbers is 32. Two times the smaller number is 10 more than the larger number. Determine the two numbers.

(5) 11. A rectangular field has a perimeter of 4080 m. The length is 240 m less than twice the width. Determine the length and width of the field. Use a final statement in your answer.

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

(6) 12. Solve the system.

$$6x - 4y = 3$$
$$9x + 8y = 1$$

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

(6) 13. Four beefburgers and three glasses of milk cost $8.50. Two beefburgers and one glass of milk cost $3.90. Determine the cost of one beefburger and the cost of one glass of milk.

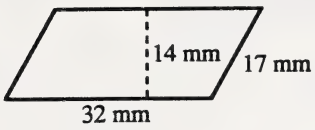
Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

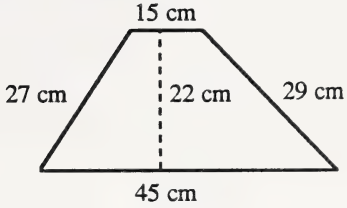
Value

(10) 14. Calculate the following areas.

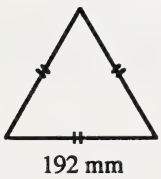
a.



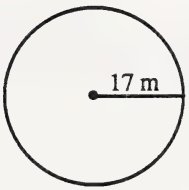
b.



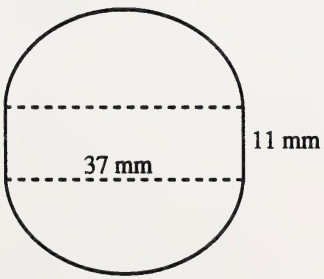
c.



d.



e.

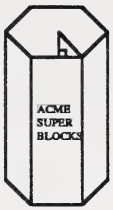


Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

(5) 15. What volume will be displaced by a regular-hexagonal prism that is used for packaging toy building blocks? Each side of the hexagon is 8 cm. The radius of the hexagon is 7 cm. The height of the container is 22 cm.



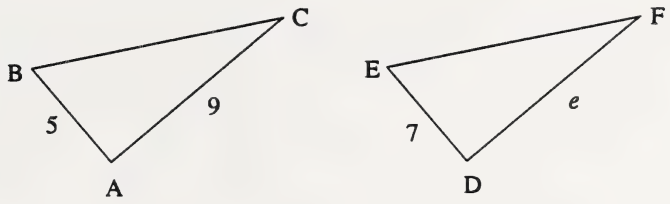
(5) 16. What is the diameter of the moon Europa if it has a surface area of 30 270 000 km^2?

Name of Student _____ Student I.D. # _____

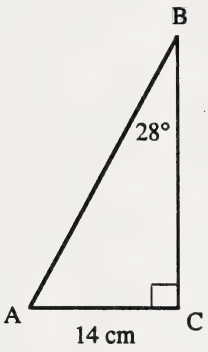
Name of School _____ Date _____

Value

(4) 17. Given $\Delta ABC \sim \Delta DEF$, determine the value of e to one decimal place.



(5) 18. Given a right triangle ABC, where $\angle B = 28°$ and side $b = 14$ cm, determine the length of the hypotenuse of the triangle. Round the final answer to two decimal places.



Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

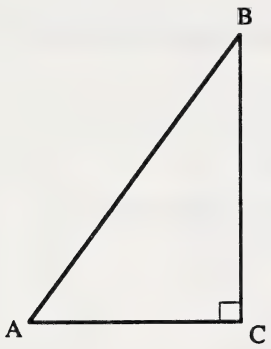
(5) 19. A post 1.8 m tall casts a shadow 2.9 m along the ground. At the same time, a building has a shadow 35.8 m long. Determine the height of the building to one decimal place.

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

(6) 20. Solve ΔABC if $\angle C = 90°$, BA = 8.7 cm, and BC = 6.3 cm. Round the angle measurements to a whole number and the side measurement to one decimal place.



Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

(5) 21. There are 15 cards of equal size. Each card has a number from 1 to 15. The cards are placed in a hat and mixed. Determine the theoretical probabilities for the following situations.

a. What is the probability of drawing a number greater than 11?

b. What is the probability of drawing an odd number?

c. What is the probability of drawing a prime number?

d. What is the probability of drawing a number greater than 6 but less than 12?

e. What is the probability of drawing a number less than 6 and greater than 10?

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

22. Use the information in the table to answer the following questions.

Math Marks	
Mark (%)	Frequency
0 - 40	4
41 - 45	6
46 - 50	5
51 - 55	7
56 - 60	8
61 - 65	12
66 - 70	13
71 - 75	12
76 - 80	9
81 - 85	8
86 - 90	2
91 - 95	2
96 - 100	1
Total	89

(1) a. What is the probability that a mark was above 90%?

(2) b. What is the probability that a mark was from 51 to 75?

(2) c. A mark over 50 but less than 66 is a C standing. What percent of the students receive C's?

(2) d. If 118 students wrote the exam, determine how many students would receive a mark from 66 to 80%.

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

23. An office worker has ten different hats. This worker decides to randomly select a hat to wear to the office each day for the five-day work week. Use a simulation to answer the following questions. Use the random number table of 50 trials given below and let the digits 0 to 9 each represent a different hat. Each block of 5 will represent a five-day week. To answer the questions, each trial must be checked.

52097	96239	72255	86588	13395	60424	63156	92309	79545	64637
58233	53891	61336	38086	39495	37774	27949	34573	58358	54006
10884	42540	17696	86020	78016	38734	04767	49518	19256	24990
35593	83183	91244	68292	24406	06991	47542	18392	25866	76750
67050	00008	52103	59222	26126	74210	01129	15483	86292	53468

(3) a. Estimate the probability that this worker will wear a different hat each day.

(1) b. Estimate the probability that this worker will wear the same hat each day.

(1) c. Estimate the probability that this worker will wear the same hat more than once in one week.

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

TABLE OF TRIGONOMETRIC FUNCTIONS

ANGLE deg	sin	cos	tan	csc	sec	cot
0	0.0000	1.0000	0.0000	undefined	1.0000	undefined
1	0.0175	0.9998	0.0175	57.299	1.0002	57.2900
2	0.0349	0.9994	0.0349	28.654	1.0006	28.6363
3	0.0523	0.9986	0.0524	19.107	1.0014	19.0811
4	0.0698	0.9976	0.0699	14.336	1.0024	14.3007
5	0.0872	0.9962	0.0875	11.474	1.0038	11.4301
6	0.1045	0.9945	0.1051	9.5668	1.0055	9.5144
7	0.1219	0.9925	0.1228	8.2055	1.0075	8.1443
8	0.1392	0.9903	0.1405	7.1853	1.0098	7.1154
9	0.1564	0.9877	0.1584	6.3925	1.0125	6.3138
10	0.1736	0.9848	0.1763	5.7588	1.0154	5.6713
11	0.1908	0.9816	0.1944	5.2408	1.0187	5.1446
12	0.2079	0.9781	0.2126	4.8097	1.0223	4.7046
13	0.2250	0.9744	0.2309	4.4454	1.0263	4.3315
14	0.2419	0.9703	0.2493	4.1336	1.0306	4.0108
15	0.2588	0.9659	0.2679	3.8637	1.0353	3.7321
16	0.2756	0.9613	0.2867	3.6280	1.0403	3.4874
17	0.2924	0.9563	0.3057	3.4208	1.0457	3.2709
18	0.3090	0.9511	0.3249	3.2361	1.0515	3.0777
19	0.3256	0.9455	0.3443	3.0716	1.0576	2.9042
20	0.3420	0.9397	0.3640	2.9238	1.0642	2.7475
21	0.3584	0.9336	0.3839	2.7904	1.0711	2.6051
22	0.3746	0.9272	0.4040	2.6695	1.0785	2.4751
23	0.3907	0.9205	0.4245	2.5593	1.0864	2.3559
24	0.4067	0.9135	0.4452	2.4586	1.0946	2.2460
25	0.4226	0.9063	0.4663	2.3662	1.1034	2.1445
26	0.4384	0.8988	0.4877	2.2812	1.1126	2.0503
27	0.4540	0.8910	0.5095	2.2027	1.1223	1.9626
28	0.4695	0.8829	0.5317	2.1301	1.1326	1.8807
29	0.4848	0.8746	0.5543	2.0627	1.1434	1.8040
30	0.5000	0.8660	0.5774	2.0000	1.1547	1.7321
31	0.5150	0.8572	0.6009	1.9416	1.1667	1.6643
32	0.5299	0.8480	0.6249	1.8871	1.1792	1.6003
33	0.5446	0.8387	0.6494	1.8361	1.1924	1.5399
34	0.5592	0.8290	0.6745	1.7883	1.2062	1.4826
35	0.5736	0.8192	0.7002	1.7435	1.2208	1.4281
36	0.5878	0.8090	0.7265	1.7013	1.2361	1.3764
37	0.6018	0.7986	0.7536	1.6616	1.2521	1.3270
38	0.6157	0.7880	0.7813	1.6243	1.2690	1.2799
39	0.6293	0.7771	0.8098	1.5890	1.2868	1.2349
40	0.6428	0.7660	0.8391	1.5557	1.3054	1.1918
41	0.6561	0.7547	0.8693	1.5243	1.3250	1.1504
42	0.6691	0.7431	0.9004	1.4945	1.3456	1.1106
43	0.6820	0.7314	0.9325	1.4663	1.3673	1.0724
44	0.6947	0.7193	0.9657	1.4396	1.3902	1.0355
45	0.7071	0.7071	1.0000	1.4142	1.4142	1.0000

ANGLE deg	sin	cos	tan	csc	sec	cot
45	0.7071	0.7071	1.0000	1.4142	1.4142	1.0000
46	0.7193	0.6947	1.0355	1.3901	1.4396	0.9657
47	0.7314	0.6820	1.0724	1.3673	1.4663	0.9325
48	0.7431	0.6691	1.1106	1.3456	1.4945	0.9004
49	0.7547	0.6561	1.1504	1.3250	1.5243	0.8693
50	0.7660	0.6428	1.1918	1.3054	1.5557	0.8391
51	0.7771	0.6293	1.2349	1.2868	1.5890	0.8098
52	0.7880	0.6157	1.2799	1.2690	1.6243	0.7813
53	0.7986	0.6018	1.3270	1.2521	1.6616	0.7536
54	0.8090	0.5878	1.3764	1.2361	1.7013	0.7265
55	0.8192	0.5736	1.4281	1.2208	1.7435	0.7002
56	0.8290	0.5592	1.4826	1.2062	1.7883	0.6745
57	0.8387	0.5446	1.5399	1.1924	1.8361	0.6494
58	0.8480	0.5299	1.6003	1.1792	1.8871	0.6249
59	0.8572	0.5150	1.6643	1.1666	1.9416	0.6009
60	0.8660	0.5000	1.7321	1.1547	2.0000	0.5774
61	0.8746	0.4848	1.8040	1.1434	2.0627	0.5543
62	0.8829	0.4695	1.8807	1.1326	2.1301	0.5317
63	0.8910	0.4540	1.9626	1.1223	2.2027	0.5095
64	0.8988	0.4384	2.0503	1.1126	2.2812	0.4877
65	0.9063	0.4226	2.1445	1.1034	2.3662	0.4663
66	0.9135	0.4067	2.2460	1.0946	2.4586	0.4452
67	0.9205	0.3907	2.3559	1.0864	2.5593	0.4245
68	0.9272	0.3746	2.4751	1.0785	2.6695	0.4040
69	0.9336	0.3584	2.6051	1.0712	2.7904	0.3839
70	0.9397	0.3420	2.7475	1.0642	2.9238	0.3640
71	0.9455	0.3256	2.9042	1.0576	3.0716	0.3443
72	0.9511	0.3090	3.0777	1.0515	3.2361	0.3249
73	0.9563	0.2924	3.2709	1.0457	3.4203	0.3057
74	0.9613	0.2756	3.4874	1.0403	3.6280	0.2867
75	0.9659	0.2588	3.7321	1.0353	3.8637	0.2679
76	0.9703	0.2419	4.0108	1.0306	4.1336	0.2493
77	0.9744	0.2250	4.3315	1.0263	4.4454	0.2309
78	0.9781	0.2079	4.7046	1.0223	4.8097	0.2126
79	0.9816	0.1908	5.1446	1.0187	5.2408	0.1944
80	0.9848	0.1736	5.6713	1.0154	5.7588	0.1763
81	0.9877	0.1564	6.3138	1.0125	6.3925	0.1584
82	0.9903	0.1392	7.1154	1.0098	7.1853	0.1405
83	0.9925	0.1219	8.1443	1.0075	8.2055	0.1228
84	0.9945	0.1045	9.5144	1.0055	9.5668	0.1051
85	0.9962	0.0872	11.4301	1.0038	11.474	0.0875
86	0.9976	0.0698	14.3007	1.0024	14.336	0.0699
87	0.9986	0.0523	19.0811	1.0014	19.107	0.0524
88	0.9994	0.0349	28.6363	1.0006	28.654	0.0349
89	0.9998	0.0175	57.2900	1.0002	57.299	0.0175
90	1.0000	0.0000	undefined	1.0000	undefined	0.0000

Teacher Questionnaire

Course name & number _____ Teacher's name _____

Teacher's area of expertise _____ School name _____

Date _____

Design

1. The modules follows a definite systematic design. Did you find it easy to follow?

   Yes No If no, explain.

   _____

   _____

2. Did your observations reveal that the students found the design easy to follow?

   Yes No If no, explain.

   _____

   _____

3. Did you find the Learning Facilitator's Manual helpful?

   Yes No If no, explain.

   _____

   _____

4. Part of the design involves stating the objectives in student terms. Do you feel this helped the students understand what they were going to learn?

   Yes No If no, explain.

   _____

   _____

5. The activities include self-assessment questions. These questions are to help clarify and reinforce the instructional materials. Did students use the answers effectively?

   Yes No If no, explain.

   _____

   _____

6. Did the two streams of Follow-up Activites (Extra-help and Enrichment) prove to be helpful?

   Yes No If no, explain.

   _____

   _____

7. Were students motivated to try these Follow-up Activities on their own?

   Yes No If no, give details.

   _____

   _____

8. Were the assignments clear?

   Yes No If no, give details.

   _____

   _____

9. Were the assignments appropriate?

   Yes No If no, give details.

   _____

   _____

10. Did you fax assignments?

Yes No

11. If you did fax, did you get satisfactory results from using this procedure?

Yes No If no, give details.

_____

_____

_____

Instruction

1. Did you find the instruction clear?

Yes No If no, give details.

_____

_____

2. Did your observations reveal that the students found the instruction interesting?

Yes No If no, give details.

_____

_____

3. Did you find the instruction adequate?

Yes No If no, give details.

_____

_____

4. Was the reading level appropriate?

Yes No If no, give details.

_____

_____

5. Was the work load adequate?

Yes No If no, give details.

_____

_____

6. Was the content accurate and current?

Yes No If no, give details.

_____

_____

7. Did the content flow consistently and logically?

Yes No If no, give details.

_____

_____

8. Was the transition between booklets smooth?

Yes No If no, give details.

_____

_____

9. Was the transition between print and media smooth?

Yes No If no, give details.

Additional Comments

When you have completed this questionnaire, please mail it to the following address.

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Alberta Correspondence School
Box 4000
Barrhead, Alberta
T0G 2P0

Mathematics 23

9MA23T20

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Producer

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